DOCUMENT RESUME

ED 239 835 SE 041 718

TITLE Spirolaterals.
INSTITUTION National Council of Teachers of Mathematics, Inc., Reston, Va.
PUB DATE Sep 83
NOTE 5p.
PUB TYPE Guides - Classroom Use - Materials (For Learner) (051) -- Collected Works - Serials (022)
JOURNAL CIT NCTM Student Math Notes; p1-4 Sep 1983

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Computer Programs; Elementary Secondary Education; *Geometric Constructions; Instructional Materials; Learning Activities; *Mathematics Instruction
IDENTIFIERS LOGO Programing Language; PF Project; *Spirolaterals

ABSTRACT

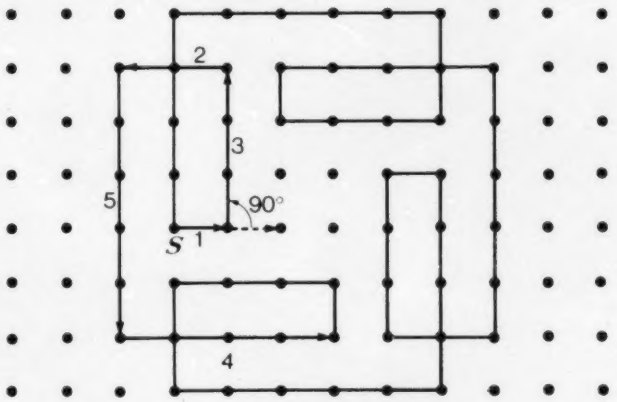
Mathematics activities involving the construction of spirolaterals are presented in this issue of "Student Math Notes." Spirolaterals are figures in which sequences of line segments of differing lengths, connected by angles of a constant size, are drawn repeatedly to create a geometric design. The activities presented involve the construction of various spirolaterals using square dot paper, triangular (isometric) dot paper, and the LOGO programming language. (DC)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

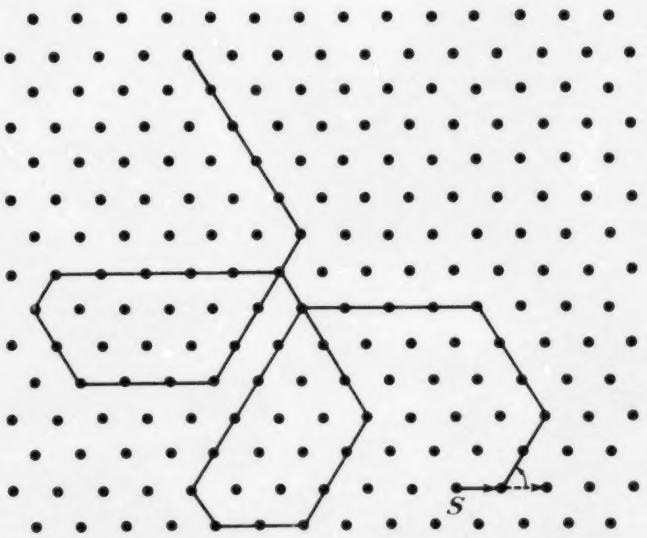
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
STUDENT math notes
SEPTEMBER 1983

ED239835

Spirolaterals



Start at point **S** and follow the arrows to trace out this design. It is made using a turning angle of 90 degrees in a counterclockwise direction, and the number sequence 1, 3, 2, 5, 4. The sequence, which represents the lengths of the line segments used, is repeated four times. At that point the figure becomes closed, since the starting and ending points coincide. Note how the turning angle at each point lies to the right of the line segment to be drawn next.



U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
✓ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Charles R. Hucka

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE041718

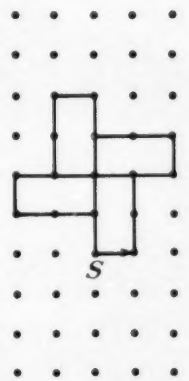
Start at point **S**, and turn in a counterclockwise direction. Can you determine the turning angle and number sequence for this design? Complete the figure so that the starting and ending points coincide.

Mathematics . . . as the Worm Turns

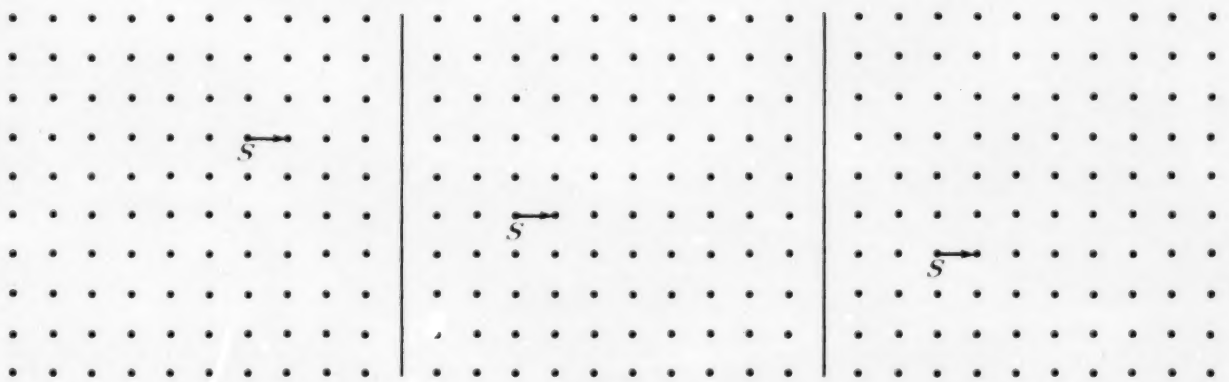
Spirolaterals follow a certain kind of mathematical pattern. They originated from the study of feeding patterns of prehistoric worms. Michael Paterson and John Conway of Great Britain, and Michael Beeler of the USA, carried out computer simulations of a variety of idealized worm eating patterns. In the early 1970s, Frank Odds, a British biochemist, proposed a set of rules for generating similar patterns, which he named *spirolaterals*. The two figures on the first page, when completed, form spirolaterals; the first is drawn on square dot paper, the second on triangular dot paper.

Spirolaterals on Square Dot Paper

To make a spirolateral, begin by choosing a starting point **S**, a sequence of numbers, a turning angle, and a direction—clockwise (CW) or counterclockwise (CCW). Alternate drawing segments with lengths matching the numbers in your sequence and using the turning angle chosen. Repeat the sequence as many times as needed to make the starting and ending points coincide, or until you feel sure your design will not close. The completed design shown at the right results from the sequence 1, 2, 3 and a turning angle of 90° in the CCW direction. This sequence is repeated four times to make the starting and ending points coincide.



Construct each spirolateral below from the given sequence and a turning angle of 90° CCW.



1. On a separate sheet make spirolaterals for these sequences: 1 through 7, 1 through 8, 1 through 9, and 1 through 10. Use a turning angle of 90° CCW.
2. Which sequences produce spirolaterals that do not close?
3. Which sequences must be repeated four times to make the starting and ending points the same? Which close with fewer or more than four repetitions of the sequence?
4. Are any of the spirolaterals you made symmetrical? Which ones, and in what way?

Spirolaterals on Triangular, or Isometric, Dot Paper

The spirolateral you were asked to complete on the cover is drawn on triangular dot paper. This is also called isometric dot paper. If you draw line segments connecting adjacent dots, as in figure 1, then equilateral triangles with 60-degree angles are formed. This paper is useful for making spirolaterals with turning angles of 60° or 120°. Figure 2 is a spirolateral for the sequence 1, 2 and a turning angle of 60° CCW. Figure 3 is a spirolateral for the sequence 1, 2 and a turning angle of 120° CCW. Each sequence is repeated three times.

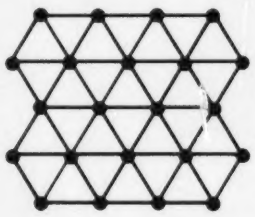


Figure 1

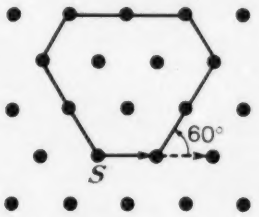


Figure 2

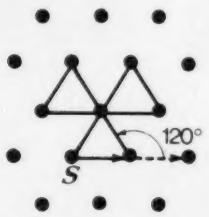
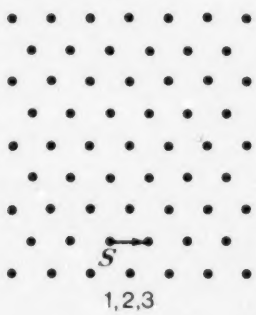
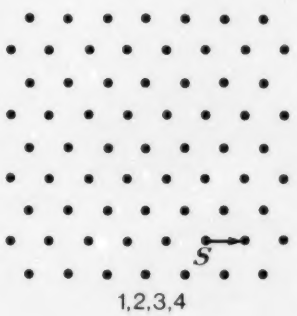


Figure 3

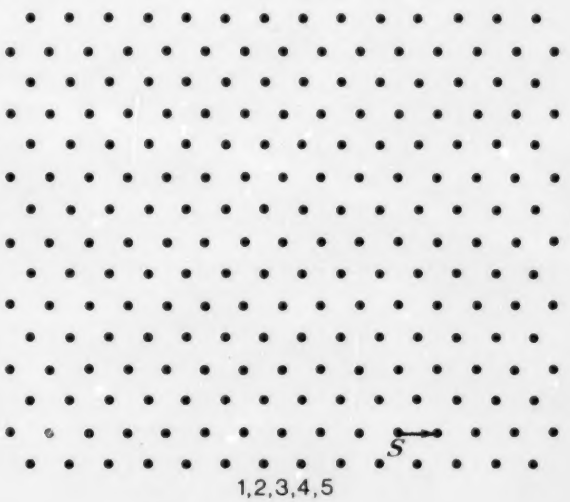
Construct each spirolateral below from the given sequence and a turning angle of 60° CCW.



1,2,3

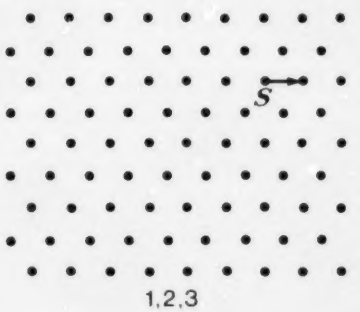


1,2,3,4

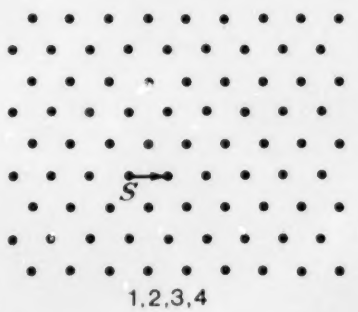


1,2,3,4,5

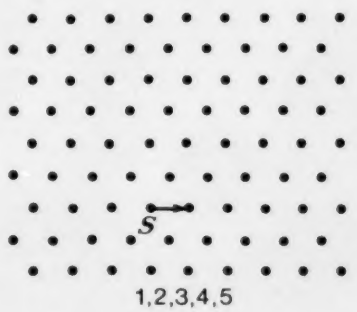
Construct each spirolateral in the next set from the given sequence and a turning angle of 120° CCW.



1,2,3



1,2,3,4



1,2,3,4,5

1. On a separate sheet make spirolaterals for these sequences: 1 through 6, 1 through 7, 1 through 8, 1 through 9, and 1 through 10. Use turning angles of 60° and 120° CCW for each one.
2. Which sequences produce spirolaterals that do not close?
3. What repetition patterns can you find?
4. Are any of the spirolaterals you made symmetrical? Which ones, and in what way?

Spirolateral Square Designs

The five-number sequence 1, 4, 1, 5, 6 with turning angle 90° CCW, when repeated four times, produces a square design, or dissection of a square. Try it out and see. But not all sequences of five numbers with turning angle 90° CCW result in square designs. For example, the five-number sequence 1, 3, 2, 5, 4 on the cover does not make a square design.

On a separate sheet, test each sequence below to see which ones give a square design when repeated four times using a turning angle of 90° CCW.

a. 4, 1, 4, 5, 2 *b.* 1, 1, 3, 1, 5 *c.* 3, 2, 3, 4, 1
d. 4, 5, 4, 3, 1 *e.* 5, 2, 3, 2, 5 *f.* 2, 1, 2, 5, 3

Construct some of your own sequences of five numbers that yield square designs.

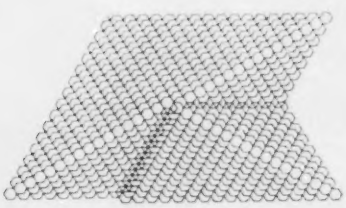
Spirolaterals and LOGO

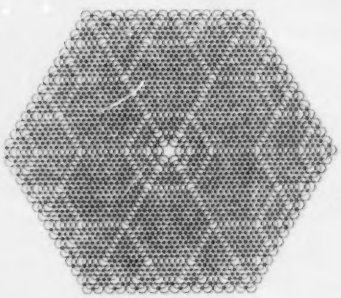
As some of you may know, LOGO is a computer language currently available on many microcomputers. One of its best features is the ease with which graphics can be produced. Investigating spirolateral patterns using the LOGO language is an exciting activity. Using the microcomputer, you can easily explore spirolaterals with turning angles other than 60°, 90°, and 120°. To get started try this procedure:

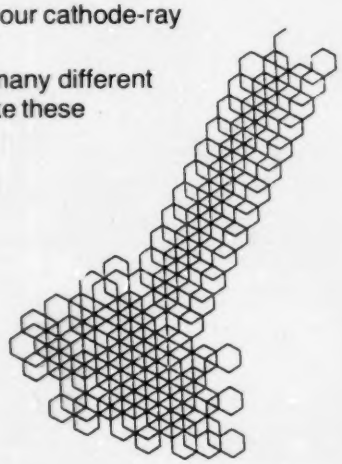
```
TO SPIRO        (Note: RT and FD are abbreviations for RIGHT and FORWARD.)
RT 90
REPEAT 4 [FD 10 RT 90 FD 30 RT 90 FD 20 RT 90 FD 50 RT 90 FD 40 RT 90]
HOME
END
```

Once you have this procedure in your workspace and you type SPIRO, the result on your cathode-ray tube should be a figure that looks like the first design on the cover.

Experiment with SPIRO by changing the angle, or the numbers, or both. There are many different alternatives to consider! Perhaps you will be able to develop complex spirolaterals like these isometric worm patterns generated by Michael Beeler and the computer at the MIT Artificial Intelligence Laboratory.







Bibliography

Gardner, Martin. "Mathematical Games." *Scientific American* 229 (November 1973): 116–23.

Kenney, Margaret, and Stanley Bezuszka. "Square Spirolaterals." *Mathematics Teaching* 95 (June 1981): 26–27.

Odds, Frank C. "Spirolaterals." *Mathematics Teacher* 66 (February 1973): 121–24.

Schwandt, Alice Kaseberg. "Spirolaterals: Advanced Investigations from an Elementary Standpoint." *Mathematics Teacher* 72 (March 1979): 166–69.

NCTM STUDENT MATH NOTES is published as part of the **NEWS BULLETIN** by the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091. The five issues a year appear in September, November, January, March, and May. Pages may be reproduced for classroom use without permission.

Editor: **Evan Maletsky,** Department of Mathematics and Computer Science, Montclair State College, Upper Montclair, NJ 07043
Editorial Panel: **Pamela Coffield,** Brookstone School, 440 Bradley Park Drive, Columbus, GA 31995
Margaret Kenney, Mathematics Institute, Boston College, Chestnut Hill, MA 02167
Stephen Krulik, Department of Secondary Education, Temple University, Philadelphia, PA 19122

Printed in U.S.A.